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Three-point Green's function of massless QED in position space to lowest order

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Abstract
The transverse part of the three-point Green's function of massless QED is determined to the lowest order in position space. Taken together with the evaluation of the longitudinal part in Mitra (2008) (*J. Phys. A: Math. Theor.* **41** 315401), this gives a relation for QED which is analogous to the star-triangle relation. We relate our result to conformal-invariant three-point functions.

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1. Introduction

In theories of massless particles with dimensionless couplings, tree-level integrals in position space can often be evaluated exactly. The simplest example, which evaluates the three-point Green's function involving three massless scalar fields [1–3] is called the star-triangle relation (also called the uniqueness relation). A similar relation has been found for the massless Yukawa theory as well [2, 3]. For massless QED with dimensionless coupling in a general number of dimensions, the longitudinal part of the three-point function to the lowest order in position space was recently determined in [3]. In this paper, we determine the transverse part. Taken together, these two results give a relation analogous to the star-triangle relation.

The star-triangle relation, being exact, is very useful in higher order calculations in perturbation theory [4–6]. The considerations of the present work are directly relevant for massless QED_3, as this theory has a dimensionless effective coupling constant in the infrared [7, 8]. So, application to higher orders of this theory is the main motivation for our work.

Moreover, as explained in the introduction of [3], our result is expected to exhibit conformal-invariant structures. The investigation of conformal transformation of the gauge field in [9, 10] led to the conformal-invariant structures of the vertex function of QED. It is

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therefore of interest to find out whether an explicit lowest order calculation brings out these structures.

The paper is organized as follows. In section 2, the transverse part of the three-point function of QED is evaluated. In section 3, we give the total result, including the longitudinal part. In section 4, we relate our result to conformal-invariant three-point functions. Our conclusions are presented in section 5. In the appendix, we perform a check of our result for the case of $D = 4$.

2. Transverse part of the three-point Green's function

As in [3], we will use the operator algebraic method due to Isaev [11] in which one reduces Feynman integrals to products of position and momentum operators $\hat{q}_i$ and $\hat{p}_i$ taken between position eigenstates. As explained in section 2 of [3], this method involves starting from the '$\hat{p}\hat{q}\hat{p}$' form and passing to the '$\hat{q}\hat{p}\hat{q}$' form. In what follows, we proceed in the same way as in section 4 of [3], where we evaluated the longitudinal part of the three-point function with the fermion propagator $\not{p}/p^2$ and photon scale dimension $\Delta = 1$ [9, 10] (this corresponds to both QED_4 and the infrared limit of massless QED_3). We will use the regularized scale dimensions given by equation (16) of [3]:

$$\Delta = 1 + \epsilon, \qquad d = \frac{D - 1 - \epsilon}{2}. \tag{1}$$

Here d denotes the fermion scale dimension and D the number of (Euclidean) dimensions. The starting '$\hat{p}\hat{q}\hat{p}$' form for the transverse part of $\langle\psi(x)\bar{\psi}(0)A_k(y)\rangle$ is then

$$\Gamma_k^t = \gamma_i\gamma_l\gamma_j\hat{p}_i\hat{p}^{-2-\epsilon}\hat{q}_j\hat{q}^{-D+\epsilon}\hat{p}^{-D+2+2\epsilon}\mathcal{P}_{lk}, \tag{2}$$

where $\mathcal{P}_{lk}$ is the transverse projection operator:

$$\mathcal{P}_{lk} = \delta_{lk} - \hat{p}_l\hat{p}_k\hat{p}^{-2}. \tag{3}$$

Equation (2) is the counterpart of equation (17) of [3], which gives the longitudinal part. Thus, for $\epsilon = 0$, equation (2) gives

$$\langle x|\Gamma_k^t|y\rangle = \mathrm{i}\frac{(D-2)}{(2\pi)^D}\int \mathrm{d}^D z \frac{\not{x} - \not{z}}{|x-z|^D}\gamma_l\frac{\not{z}}{|z|^D}\left(\delta_{lk} - \frac{\partial_l^y\partial_k^y}{(\partial^2)^y}\right)\frac{1}{|y-z|^2}. \tag{4}$$

Equation (4) is the counterpart of equation (15) of [3].

Let us now recall from [3] that in proceeding from equation (2) to the '$\hat{q}\hat{p}\hat{q}$' form, one has to move $\hat{q}_i$ (or $\hat{p}_i$) through powers of $\hat{p}^2$ (or $\hat{q}^2$) by using $[\hat{q}_i, \hat{p}^{2\alpha}] = \mathrm{i}2\alpha\hat{p}^{2\alpha-2}\hat{p}_i$ (or $[\hat{p}_i, \hat{q}^{2\alpha}] = -\mathrm{i}2\alpha\hat{q}^{2\alpha-2}\hat{q}_i$). One also has to make use of the key relation

$$\hat{p}^{-2\alpha}\hat{q}^{-2(\alpha+\beta)}\hat{p}^{-2\beta} = \hat{q}^{-2\beta}\hat{p}^{-2(\alpha+\beta)}\hat{q}^{-2\alpha}, \tag{5}$$

which is the star-triangle relation in the operator form, at some intermediate stage. Using $\{\gamma_j, \gamma_l\} = 2\delta_{jl}$ in equation (2), one can write Γ_k^t as the sum of two terms, say, $\Gamma_k^{t(1)}$ and $\Gamma_k^{t(2)}$. One term is

$$\Gamma_k^{t(1)} = 2\gamma_i\hat{p}_i\hat{p}^{-2-\epsilon}\hat{q}^{-D+\epsilon}\hat{q}_l\hat{p}^{-D+2+2\epsilon}\mathcal{P}_{lk}. \tag{6}$$

Now $\hat{q}_l$ can be taken through $\hat{p}^{-D+2+2\epsilon}$ to the right without generating an extra term, since the commutator, being proportional to $\hat{p}_l$, is annihilated by $\mathcal{P}_{lk}$. Next, use equation (5) to obtain

$$\Gamma_k^{t(1)} = 2\gamma_i\hat{p}_i\hat{q}^{-D+2+2\epsilon}\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon}\hat{q}_l\mathcal{P}_{lk}. \tag{7}$$

Finally, one takes $\hat{p}_i$ through $\hat{q}^{-2-\epsilon}$ to the left to arrive at

$$\Gamma_k^{t(1)} = 2\gamma_i(\hat{q}^{-D+2+2\epsilon}\hat{p}_i + \mathrm{i}(D-2-2\epsilon)\hat{q}^{-D+2\epsilon}\hat{q}_i)\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon}\hat{q}_l\mathcal{P}_{lk}. \tag{8}$$

The other term is

$$\Gamma_k^{t(2)} = -\gamma_i\gamma_j\gamma_l\hat{p}_i\hat{p}^{-2-\epsilon}\hat{q}_j\hat{q}^{-D+\epsilon}\hat{p}^{-D+2+2\epsilon}\mathcal{P}_{lk}. \tag{9}$$

Except for γ_l and $\mathcal{P}_{lk}$, this is the '$\hat{p}\hat{q}\hat{p}$' form for the vertex of the massless Yukawa theory (see equation (5) of [3]). Therefore, following exactly the same steps as for that case, equation (9) leads to

$$\Gamma_k^{t(2)} = -\gamma_i\gamma_j\gamma_l\hat{p}_i(\hat{q}_j\hat{p}^{-2-\epsilon} + \mathrm{i}(2+\epsilon)\hat{p}^{-4-\epsilon}\hat{p}_j)\hat{q}^{-D+\epsilon}\hat{p}^{-D+2+2\epsilon}\mathcal{P}_{lk} \tag{10}$$

$$= -(\gamma_i\gamma_j\gamma_l\hat{p}_i\hat{q}_j\hat{q}^{-D+2+2\epsilon}\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon} + \mathrm{i}(2+\epsilon)\gamma_l\hat{q}^{-D+2+2\epsilon}\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon})\mathcal{P}_{lk} \tag{11}$$

$$= (-\gamma_i\gamma_j\gamma_l\hat{q}_j\hat{q}^{-D+2+2\epsilon}\hat{p}_i\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon} + \mathrm{i}\epsilon\gamma_l\hat{q}^{-D+2+2\epsilon}\hat{p}^{-D+\epsilon}\hat{q}^{-2-\epsilon})\mathcal{P}_{lk}. \tag{12}$$

It may be noted that though the matrix element of $\hat{p}^{-4-\epsilon}$ in position space is infrared divergent for $D \leqslant 4$, in equation (10) this operator comes multiplied with $\gamma_i\gamma_j\hat{p}_i\hat{p}_j$ and gives $\hat{p}^{-2-\epsilon}$. Therefore, it is legitimate to use equation (5) in the following step (the star-triangle relation, as given in equation (1) of [3], holds only when $\delta_i > 0$; see [2]).

Now,

$$\langle x|\Gamma_k^t|y\rangle = \left(\delta_{kl} - \frac{\partial_k^y\partial_l^y}{(\partial^2)^y}\right)\langle x|\big(\Gamma_l^{t(1)'} + \Gamma_l^{t(2)'}\big)|y\rangle, \tag{13}$$

where $\Gamma_l^{t(1)'}$ and $\Gamma_l^{t(2)'}$ are given by the right-hand sides of equations (8) and (12), *omitting* $\mathcal{P}_{lk}$. It is important to note that due to the presence of the transverse projection operator on the right-hand side of equation (13), *if any term in the matrix element following it can be expressed as a derivative with respect to y_l, that term will not contribute.* The evaluation of the matrix element is to be done using the matrix elements given in the appendix of [3]. It is found that in three of the resulting terms, the diverging $\Gamma(\epsilon/2)$ comes multiplied by ϵ. So taking $\epsilon \to 0$ gives finite results for these terms in a straightforward way. However, there is also the contribution (this is the second term on the right-hand side of equation (8), without $\mathcal{P}_{lk}$, taken between $\langle x|$ and $|y\rangle$)

$$2\mathrm{i}\frac{(D-2-2\epsilon)\Gamma(\epsilon/2)}{\pi^{D/2}2^{D-\epsilon}\Gamma(D/2-\epsilon/2)}\frac{\not{x}}{|x|^{D-2\epsilon}}\frac{1}{|x-y|^{\epsilon}}\frac{y_l}{|y|^{2+\epsilon}}. \tag{14}$$

For $\epsilon \to 0$, there is an apparent $O(1/\epsilon)$ singularity. But the y-dependence of this singular term is just $y_l/|y|^2$, which equals $(\partial/\partial y_l)\ln|y|$, and so the singular term will not contribute. There is still a finite contribution from (14), which has to be taken into account. The y-dependences in this contribution will involve only $y_l/|y|^2$, and also $y_l/|y|^2$ times $\ln(|x-y||y|/|x|^2)$. The only surviving contribution is $(y_l/|y|^2)\ln|x-y|$. (Since $(y_l/|y|^2)\ln|y| = (1/2)(\partial/\partial y_l)(\ln|y|)^2$, it drops out.) Putting everything together, we arrive at

$$\langle x|\big(\Gamma_l^{t(1)'} + \Gamma_l^{t(2)'}\big)|y\rangle = \frac{\mathrm{i}}{\pi^{D/2}2^{D-1}\Gamma(D/2)}\left(\frac{(\not{x}-\not{y})\gamma_l\not{y}}{|x|^{D-2}|x-y|^2|y|^2} - 2(D-2)\frac{\ln|x-y|\not{x}y_l}{|x|^D|y|^2}\right). \tag{15}$$

The right-hand sides of equations (4), (13) and (15) taken together give the result for the transverse part of the three-point function. We reiterate that we have the logarithm of the dimensionful object $|x-y|$ in equation (15) only because we have dropped all terms annihilated by the transverse projection operator.

3. Total result for three-point Green's function

Adding the longitudinal part given by equation (28) of [3], the result for $\langle T(\psi(x)\bar{\psi}(0)A_k(y)\rangle$ to the lowest order is

$$\begin{aligned}\int \mathrm{d}^D z \frac{\not{x}-\not{z}}{|x-z|^D}\gamma_l \frac{\not{z}}{|z|^D}\left(\delta_{kl}-(1-\eta)\frac{\partial_k^y\partial_l^y}{(\partial^2)^y}\right)\frac{1}{|y-z|^2} \\ = \frac{2\pi^{D/2}}{(D-2)\Gamma(D/2)}\left[\left(\delta_{kl}-\frac{\partial_k^y\partial_l^y}{(\partial^2)^y}\right)\right. \\ \times\left(\frac{(\not{x}-\not{y})\gamma_l\not{y}}{|x|^{D-2}|x-y|^2|y|^2}-2(D-2)\frac{\ln|x-y|\not{x}y_l}{|x|^D|y|^2}\right) \\ \left.+\eta\frac{\not{x}}{|x|^D}\left(\frac{(x-y)_k}{|x-y|^2}+\frac{y_k}{|y|^2}\right)\right].\end{aligned} \tag{16}$$

Here η is the gauge parameter. Letting $x = x_1 - x_2$ and $y = x_3 - x_2$, and also changing to a new integration variable x_4 defined by $z = x_4 - x_2$, one can immediately write this relation in a manifestly translation-invariant form, as was done for the relations in [3]. The right-hand side then depends on the differences of the three external coordinates, taken in pairs. Thus we have a relation for massless QED similar to the star-triangle relation, with not only the structure functions but also their coefficients exactly determined. That the transverse projection operator is present on the right-hand side of equation (16) is not a cause for concern, because its presence is actually useful for relating to the conformal-invariant structures given in the literature, and also for calculation at higher orders (as the orthogonality of the transverse part and the remaining longitudinal part is manifested).

In section 4, we compare equation (16) with the structure functions given from general considerations of conformal invariance in [9, 10]. We find that the logarithmic term in the transverse part does not appear in these references. Because of this, we perform a check in the appendix to confirm the presence of this term. It is based on the observation that $(\partial^2)^y$ acting on the right-hand side of equation (16) can be readily calculated. Also, for $\eta = 1$, the y-dependence of the left-hand side is just $1/|y-z|^2$ and for $D = 4$, $(\partial^2)^y$ acting on this gives a delta function; so $(\partial^2)^y$ acting on the left-hand side can be evaluated as well. The two resulting expressions are then found to match only if the logarithmic term is present.

4. Relation to conformal-invariant functions

Of the transformations associated with the conformal algebra, invariance (or covariance) of our result under translation, (Euclidean) rotation and scaling is obvious. Our concern will therefore be with the effect of the coordinate inversion R: $(Rx)_\mu \equiv x_\mu/x^2$. We first summarize some results on R which will be directly useful for our purpose.

A conformal vector with scale dimension Δ transforms under R as [12]

$$A_k(x) \to U A_k(x) = |x|^{-2\Delta}(\delta_{kl} - 2x_k x_l/x^2)A_l(Rx). \tag{17}$$

With this transformation law, the two conformal-invariant structures for $\langle\psi(x)\bar{\psi}(0)A_k(y)\rangle$ are (d being the scale dimension of the fermion)

$$C_{1k}^{d,\Delta}(x,y) = \frac{(\not{x}-\not{y})\gamma_k\not{y}}{|x|^{2d-\Delta}|x-y|^{\Delta+1}|y|^{\Delta+1}}, \tag{18}$$

$$C_{2k}^{d,\Delta}(x,y) = \frac{\not{x}}{|x|^{2d-\Delta+2}|x-y|^{\Delta-1}|y|^{\Delta-1}}\left(\frac{(x-y)_k}{|x-y|^2}+\frac{y_k}{|y|^2}\right) \tag{19}$$

$$= \frac{\not{x}}{|x|^{2d-\Delta+2}|x-y|^{\Delta-1}|y|^{\Delta-1}} \partial_k^y \ln \frac{|y|}{|x-y|}. \tag{20}$$

(See [13]; the structures for $\Delta = 1$ are given in [9, 10].)

For $\Delta = 1$, equation (17) leads to a conformal-invariant propagator without a transverse part. In order to accommodate the usual covariant-gauge propagator (as used by us), equation (17) can be modified to the following *new* transformation law [9, 10]:

$$A_k(x) \to \tilde{U} A_k(x) = [(1-P)U(1-P) + UP]A_k(x). \tag{21}$$

Here $P = \partial_k \partial_l / \partial^2$ is the longitudinal projection operator. The new conformal-invariant structures for $\langle \psi(x)\bar{\psi}(0)A_k(y)\rangle$ are then [9, 10] $C_{2k}^{d,\Delta=1}(x,y)$, which is purely longitudinal in y (see equation (20)), and $\left(\delta_{kl} - \partial_k^y \partial_l^y / (\partial^2)^y\right) C_{1l}^{d,\Delta=1}(x,y)$, which is (manifestly) transverse. On putting $d = (D-1)/2$, we see that *these are precisely the terms present in our result of equation (16), except for the logarithmic term.*

To understand the logarithmic term, we first note that this term, which came from $\left(\delta_{kl} - \partial_k^y \partial_l^y / (\partial^2)^y\right)$ operating on expression (14), also emerges from $\left(\delta_{kl} - \partial_k^y \partial_l^y / (\partial^2)^y\right)$ operating on

$$2\mathrm{i} \frac{(D-2-2\epsilon)\Gamma(\epsilon/2)}{\pi^{D/2} 2^{D-\epsilon}\Gamma(D/2-\epsilon/2)} \frac{\not{x}}{|x|^{D-2\epsilon}} \frac{1}{|y|^{\epsilon}} \frac{(x-y)_l}{|x-y|^{2+\epsilon}}. \tag{22}$$

The reason is that, after taking $\epsilon \to 0$ and discarding all terms which are derivatives with respect to y_l, the surviving term has the y-dependence $\ln|y|(x-y)_l/|x-y|^2$, and this is the same as $\ln|x-y|y_l/|y|^2$ (upto $(\partial/\partial y_l)(\ln|y|\ln|x-y|)$). Therefore, the average of expressions (14) and (22) also gives our logarithmic term. Comparing with equation (19), we find that this average is $C_{2l}^{d,\Delta}(x,y)$ *with d and Δ given by the regularized scale dimensions of equation* (1).

So the logarithmic term arises in the following way. The function $C_{2k}^{d,\Delta}(x,y)$ is longitudinal for $\Delta = 1$, but for $\Delta = 1+\epsilon$, it develops an $O(\epsilon)$ part which is not longitudinal and therefore not annihilated by the transverse projection operator. Because of the presence of a coefficient of $O(1/\epsilon)$, this leads to a contribution which survives the limit $\epsilon \to 0$.

We now briefly address the question: is the logarithmic term invariant under the new transformation law given in equation (21)? To answer this, let us first go through the demonstration of invariance of a transverse three-point function, say $\Gamma_k \equiv (1-P)C_k$, under equation (21) for $\Delta = 1$. This function transforms to $\Gamma'_k = (1-P)U^{\Delta=1}(1-P)C_k$, since $P(1-P) = 0$ and $(1-P)^2 = 1-P$. (We do not write the transformation of the spinors, which can be taken into account trivially.) Next, using $PU^{\Delta=1}P = U^{\Delta=1}P$ (see [9]), we get $\Gamma'_k = (1-P)U^{\Delta=1}C_k$. Finally, if C_k satisfies $U^{\Delta=1}C_k = C_k$, we arrive at $\Gamma'_k = \Gamma_k$. But $C_{2k}^{\Delta=1+\epsilon}$ satisfies $U^{\Delta=1+\epsilon}C_k = C_k$, and since $PU^{\Delta=1+\epsilon}P = U^{\Delta=1+\epsilon}P$ does not hold, the demonstration outlined above cannot be carried out for $\Gamma_k = (1-P)C_{2k}^{\Delta=1+\epsilon}$. A direct calculation with the logarithmic term also confirms non-invariance under equation (21).

This non-invariance is, however, not an artifact of the regularization of the scale dimensions, as the check performed in the appendix does not use the regularization at any stage. The possible reason for this non-invariance is that invariance under the new transformation given by equation (21) can be realized only non-perturbatively and not in perturbation theory, as stated in [14].

5. Conclusion

In this work, we have completed the evaluation of the three-point Green's function of massless QED to the lowest order in position space. This has resulted in a relation which is analogous to the star-triangle relation, and can be used for calculations to higher orders in perturbation theory. The transverse part of the three-point function was found to contain a logarithmic term, and the presence of this term was checked by a calculation in $D = 4$. The relation of the various terms in our result to conformal-invariant three-point functions was explained.

Appendix

We consider the case of $D = 4$ and Feynman gauge. Let us take equation (16) with $\eta = 1$ and operate on both sides with $(\partial^2)^y$ in $D = 4$. For the left-hand side, we have

$$(\partial^2)^y \int \mathrm{d}^4 z \frac{\not{x} - \not{z}}{|x - z|^4}\gamma_k \frac{\not{z}}{|z|^4} \frac{1}{|y - z|^2} = -4\pi^2 \frac{\not{x} - \not{y}}{|x - y|^4}\gamma_k \frac{\not{y}}{|y|^4}. \tag{A.1}$$

For the right-hand side, the following equations are to be used:

$$\left((\partial^2)^y \delta_{kl} - \partial_k^y \partial_l^y\right) \frac{(\not{x} - \not{y})\gamma_l \not{y}}{|x - y|^2 |y|^2} = -\frac{4(x^2(\not{x} - \not{y})\gamma_k \not{y} - y^2 \not{x} x_k - x^2 \not{x} y_k + 2x \cdot y \not{x} y_k)}{|x - y|^4 |y|^4} \tag{A.2}$$

$$\left((\partial^2)^y \delta_{kl} - \partial_k^y \partial_l^y\right) \frac{\ln|x - y| y_l}{|y|^2} = \frac{x_k}{|x - y|^2 |y|^2} + \frac{2(x \cdot y - y^2)}{|x - y|^2 |y|^2}\left(\frac{(x - y)_k}{|x - y|^2} + \frac{y_k}{|y|^2}\right) \tag{A.3}$$

$$(\partial^2)^y \left(\frac{(x - y)_k}{|x - y|^2} + \frac{y_k}{|y|^2}\right) = -4\left(\frac{(x - y)_k}{|x - y|^4} + \frac{y_k}{|y|^4}\right). \tag{A.4}$$

We can now evaluate the action of $(\partial^2)^y$ operating on the right-hand side of equation (16). The result (with $\eta = 1$) is found to match equation (A.1).

However, if we did not have the logarithmic term on the right-hand side of equation (16), the action of $(\partial^2)^y$ on it would have produced

$$\begin{aligned} -\frac{4\pi^2}{|x|^4 |x - y|^4 |y|^4}[\{|x|^4(\not{x} - \not{y})\gamma_k \not{y} - x^2 y^2 \not{x} x_k - |x|^4 \not{x} y_k + 2x \cdot y x^2 \not{x} y_k\} \\ + \{|x - y|^4 \not{x} y_k + |y|^4 \not{x}(x - y)_k\}]. \end{aligned} \tag{A.5}$$

Here the two expressions within the curly brackets come from equations (A.2) and (A.4); thus they are the contributions from the two structure functions given in [9]. Let us now note that there is an $(x \cdot y)^2 \not{x} y_k$ term when we expand out the second expression within the curly brackets, and this term would remain uncancelled in (A.5) *even if we had two different coefficients with the two expressions*. But there is no such term from equation (A.1). Therefore, it is impossible to express the lowest-order QED three-point function in terms of only the two standard structure functions invariant under equation (21), and the logarithmic term is essential.

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